Analysis of Arbitrarily Shaped Two-Dimensional Microwave Circuits by Finite-Difference Time-Domain Method

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Abstract—**The paper presents a version of the finite-difference time-domain method adapted to the needs of S matrix calculations of microwave two-dimensional circuits. The analysis is conducted by simulating the wave propagation in the circuit terminated by matched loads and excited by a matched pulse source. Various aspects of the method's accuracy are investigated. Practical computer implementation of the method is discussed and an example of its application to an arbitrarily shaped microstrip circuit is presented. It is shown that the method in the proposed form is an effective tool of circuit analysis in engineering applications. The method is compared to two other methods used for a similar purpose, namely the contour integral method and the transmission-line matrix method.**

I. Introduction

A TWO-DIMENSIONAL circuit as understood here is a circuit in which the fields may be characterized by a scalar function $V(x, y)$ which obeys a two-dimensional wave equation

$$\nabla_{xy}^2 V(x, y, t) - \beta^2 \frac{\partial^2 V(x, y, t)}{\partial t^2} = 0 \qquad (1)$$

with proper boundary conditions.

There are many microwave circuits characterized accurately or approximately by (1). As examples we suggest stripline or microstrip junctions or resonators, and also some types of waveguide discontinuities. That is why solving this equation was investigated by many researchers. If the boundary is of complicated shape, only numerical methods can be used. These methods can be divided into two groups. The methods of the first group require a certain amount of analytical preprocessing before the particular problem may be solved by a computer. As an example we propose the Green's function method supported by segmentation techniques [1]. Another group of methods assumes that the analytical preprocessing should be practically nonexistent and that it is up to the computer to do the entire job. In this group we find the contour integral method [2], the transmission-line matrix method [3], and the finite-difference method, which will be the subject of this paper.

The finite-difference time-domain method (FD-TD) was first introduced by Yee [4] and since then has been applied by many authors. These applications, however, were concentrated in the domain of scattering [5], wave absorption [17], and accelerator physics [6]. Application of the FD-TD method to microwave circuit analysis has so far attracted little attention.

The aim of this paper is to show that the FD-TD method adapted to the needs of 2-D circuit analysis can be a strong competitor to the other mentioned methods. It can lead to a universal and effective computer program capable of solving a wide range of practical problems. The paper extends the ideas presented in [7] and [8]. It presents an FD-TD method of 2-D circuit analysis based on pulse excitation by a matched source. It discusses various aspects of the method's accuracy and presents the author's experience with its implementation on an IBM PC AT computer.

II. Outline of the FD-TD Method

In the FD-TD method instead of solving the second-order equation (1) a pair of first-order equations is solved:

$$\nabla V(x, y, t) = -L_s \frac{\partial \boldsymbol{J}(x, y, t)}{\partial t} \qquad (2)$$

$$\nabla \cdot \boldsymbol{J}(x, y, t) = -C_s \frac{\partial V(x, y, t)}{\partial t}. \qquad (3)$$

In microwave planar circuits, the variables and constants in (2) and (3) have the following interpretations: V—voltage, $\boldsymbol{J}$—surface current density, C_s—capacitance of a unitary square of the circuit, L_s—inductance of an arbitrary square of the circuit. The xy plane is divided into a set of meshes which are basically square but may have their shapes modified to match the boundary line. The coordinates of the middle of a mesh in the kth row and the lth column are denoted by x_l and y_k. Replacing the differentials in (2) and (3) by finite differences Δt and a

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yields

$$J_x\left(x_l+\frac{a}{2}, y_k, t_0+\frac{\Delta t}{2}\right) = J_x\left(x_l+\frac{a}{2}, y_k, t_0-\frac{\Delta t}{2}\right) - \left(V(x_l+a, y_k, t_0) - V(x_l, y_k, t_0)\right)\frac{\Delta t}{L_s a f_1(l,k)} \tag{4}$$

$$J_y\left(x_l, y_k+\frac{a}{2}, t_0+\frac{\Delta t}{2}\right) = J_y\left(x_l, y_k+\frac{a}{2}, t_0-\frac{\Delta t}{2}\right) - \left(V(x_l, y_k+a, t_0) - V(x_l, y_k, t_0)\right)\frac{\Delta t}{L_s a f_2(l,k)} \tag{5}$$

$$V(x_l, y_k, t_0+\Delta t) = V(x_l, y_k, t_0) - \left(J_x\left(x_l+\frac{a}{2}, y_k, t_0+\frac{\Delta t}{2}\right) - J_x\left(x_l-\frac{a}{2}, y_k, t_0+\frac{\Delta t}{2}\right) + J_y\left(x_l, y_k+\frac{a}{2}, t_0+\frac{\Delta t}{2}\right) - J_y\left(x_l, y_k-\frac{a}{2}, t_0+\frac{\Delta t}{2}\right)\right)\frac{\Delta t}{C_s a f_3(l,k)} \tag{6}$$

where $f_1(l,k)$, $f_2(l,k)$, and $f_3(l,k)$ are mesh shape functions which are equal to unity for all square meshes (that is, those inside the circuit) but adopt different values (calculated by a boundary matching procedure like that of [7]) for the meshes modified to match boundary lines.

Consecutive calculations of (4), (5), and (6) simulate the process of the wave propagation in the circuit.

III. Problems of FD-TD Analysis of Two-Dimensional Circuits

A. Modeling of Matched Loads and Sources

The most convenient way of describing a microwave linear circuit is by its S matrix. Since the FD-TD method models the energy flow in the circuit it may be used to compute the S matrix directly, provided that the matched loads of the output lines are properly modeled in the algorithm. Matching the source, although not absolutely necessary, is highly desirable, since any reflections from the input would prolong the transient response of the circuit, thus prolonging the computing time.

The absorbing boundary conditions have been investigated in [14] and [15], but the procedures developed there are not useful in our case. They cannot handle the situation of matched source and also, being designed for general absorbing conditions, they are unnecessarily complicated when applied to the case of normal incidence. That is why a different approach will be used here.

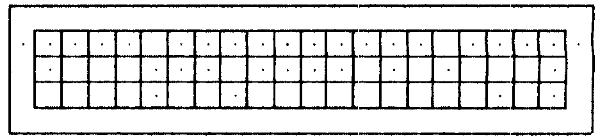

Fig. 1. A uniform transmission line as a grid of meshes.

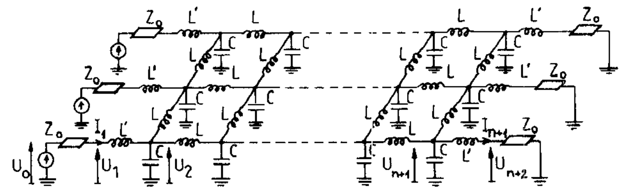

Fig. 2. Lumped circuit model of the line of Fig. 1.

Let us consider a model of a uniform transmission line of length $n=20a$ and width $w=3a$, where a is the mesh size (Fig. 1). The lumped circuit model of this line corresponding to the FD approximation is presented in Fig. 2. Propagation inside the circuit is described by (4), (5), and (6). The input and output matching is obtained by introducing in each of the rows of meshes at the input and output the following operations:

$$I_1\left(t_0+\frac{\Delta t}{2}\right) = I_1\left(t_0-\frac{\Delta t}{2}\right) - \left(V_2(t_0)-V_1(t_0)\right)\frac{\Delta t}{L'} \tag{7}$$

$$V_1(t_0+\Delta t) = V_0(t_0+\Delta t) - I_1\left(t_0+\frac{\Delta t}{2}\right)Z_0 \tag{8}$$

$$I_{n+1}\left(t_0+\frac{\Delta t}{2}\right) = I_{n+1}\left(t_0-\frac{\Delta t}{2}\right) + \left(V_{n+1}(t_0) - V_{n+2}(t_0)\right)\frac{\Delta t}{L'} \tag{9}$$

$$V_{n+2}(t_0+\Delta t) = I_{n+1}\left(t_0+\frac{\Delta t}{2}\right)Z_0 \tag{10}$$

where

$$Z_0=\sqrt{\frac{L}{C}} \qquad L'=\frac{L}{2}+L''=\frac{L}{2}-\frac{\Delta t\omega}{2a\beta}L$$

$$C=C_s a^2 \qquad L=L_s.$$

The meaning of the voltages and the currents used in (7)–(10) is explained in Fig. 2. The additional inductance L'' was introduced as a correction element to lower the matching errors caused by the fact that in the FD-TD algorithm the voltage is defined at points of time and space which are different from the points where the current is defined. Because of this it is also necessary to introduce some correction terms in the formulas for the S matrix elements. Let us assume that $V_0(\omega), V_1(\omega), \cdots$ are the complex amplitudes calculated by the Fourier transformation of $V_0(t), V_1(t), \cdots$ and $I_1(\omega), I_2(\omega), \cdots$ are the complex amplitudes calculated by the Fourier transforma-

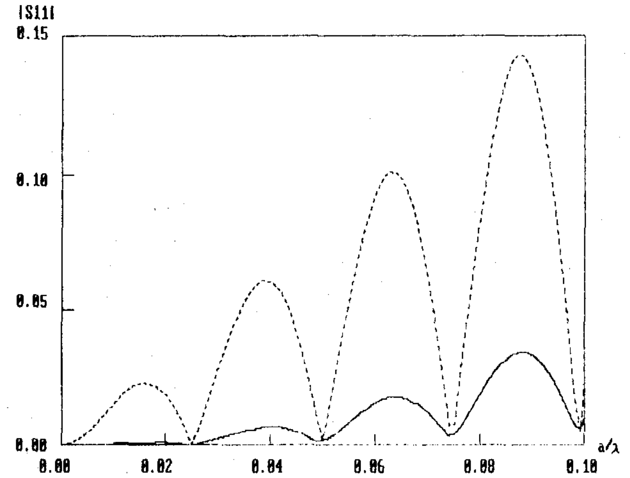

Fig. 3. Results of calculations of $|S_{11}|$ versus a/λ for the line of Fig. 1 with the special correction (continuous line) and without it (dashed line).

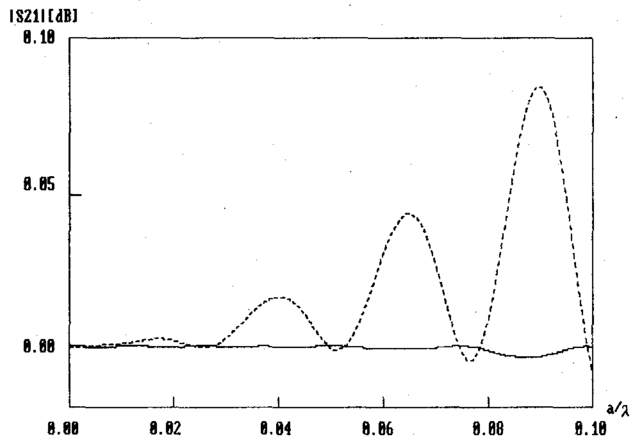

Fig. 4. Results of calculations of $|S_{21}|$ versus a/λ for the circuit of Fig. 1 with the special correction (continuous line) and without it (dashed line).

tion of $I_1(t-\Delta t/2), I_2(t-\Delta t/2), \cdots$. Resolving the circuit equations at the input and output we obtain

$$S_{21}=\frac{2V_{n+2}(\omega)\cos(\phi)}{V_0(\omega)}e^{j\phi} \tag{11}$$

$$S_{11}=\frac{Z_{\text{in}}-Z_{\text{o}}}{Z_{\text{in}}+Z_{\text{o}}} \quad \text{with } Z_{\text{in}}=\frac{V_1(\omega)}{I_1(\omega)}e^{-j\phi}-j\omega L'' \tag{12}$$

where $\phi=\omega\Delta t/2$.

Figs. 3 and 4 present the results of calculations of $|S_{11}|$ and $|S_{21}|$ versus frequency for the line of Fig. 1 with the correction terms containing ϕ and L'' (continuous line) and without them (dashed line). The frequency is characterized by the mesh size to wavelength ratio a/λ. It is seen that when the corrections are applied, the errors of matching drop to negligible values even for relatively high a/λ ratios.

Modeling of a matched source and a matched load was introduced for a uniform transmission line. It can be applied to an arbitrarily shaped circuit provided that at the input and output the circuit includes segments of uniform transmission lines long enough to ensure effective attenuation of all the width modes except the dominant one.

B. Pulse Excitation

In the approach used in [7] the circuit with zero initial values of $V(x,y,t)$ and $\boldsymbol{J}(x,y,t)$ was excited by a sinusoidal source of frequency ω. The circuit's parameters at that frequency were obtained from the steady state achieved in the circuit after a sufficiently long time. This caused the time for computing a wide-band frequency response to be very long.

Equations (4), (5), and (6) show that all the operations in the FD-TD algorithm are linear. Thus it is possible to use Fourier analysis to obtain the circuit's frequency response from the transient response. However, we need a proof that this approach will not sacrifice the accuracy of the method.

Let us consider a relation between a voltage at the input, V_0, and a voltage at the output, V_l. We may write

$$V_l(t)=\boldsymbol{L}(V_0(t))=\boldsymbol{L}'(V_0(t))+\Delta\boldsymbol{L}(V_0(t)) \tag{13}$$

where $\boldsymbol{L}$ is a linear operator of the FD-TD algorithm. $\boldsymbol{L}'$ is an operator of the transformation between input and output of the original circuit, which is linear due to the linearity of the Maxwell's equations; and $\Delta\boldsymbol{L}$ is an operator describing the error of the FD-TD method, which has to be linear due to the linearity of the two former operators.

Let us write the Fourier transform of (13):

$$V_l(\omega)=T(\omega)V_0(\omega)=(T'(\omega)+\Delta T(\omega))V_0(\omega) \tag{14}$$

where $V_0(\omega), V_l(\omega)$ are the Fourier transforms of $V_0(t)$, $V_l(t)$ and $T(\omega), T'(\omega)$, and $\Delta T(\omega)$ describe the operators $\boldsymbol{L}$, $\boldsymbol{L}'$, and $\Delta\boldsymbol{L}$ in the frequency domain. This yields

$$\frac{V_l(\omega)}{V_0(\omega)}=T(\omega)=T'(\omega)+\Delta T(\omega). \tag{15}$$

Equation (15) suggests that for any particular frequency ω, the error of the FD-TD method does not depend on the shape of the input signal and that the function $V_0(t)$ may be chosen to be the most convenient for use in the computer algorithm. However, there is one aspect of the pulse excitation which needs additional checking. The Fourier transforms in (15) are assumed to be calculated in an infinite period of time. The limited period of time assumed in any computer calculation causes additional error. Thus we must determine how much this error depends on the shape of the source.

Let us consider two types of pulses. The first is a δ pulse which in the algorithm adopts the form

$$V_0(t)=\begin{cases}0 & \text{for } t=0\\ 1 & \text{for } t=n\,\Delta t;\ n=\pm 1,\pm 2,\cdots.\end{cases} \tag{16}$$

The second is a pulse of limited spectrum approximating a δ pulse after passing through a bandpass filter of cutoff

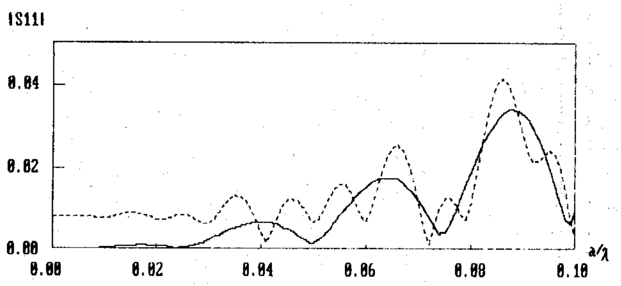

Fig. 5. $|S_{11}|$ of the circuit of Fig. 1 calculated after 200 iterations; with δ-type excitation (dashed line) and with excitation of limited spectrum (continuous line).

frequency ω_c:

$$V_0(t) = \begin{cases} 2\omega_c \Delta t/\pi & \text{for } t = 0 \\ \dfrac{2\,\Delta t \sin(\omega_c t)}{\pi t} & \text{for } t = m\,\Delta t;\ m = \pm 1, \pm 2, \pm 3, \cdots, \pm M \\ 0 & \text{for } t = m\,\Delta t;\ |m| > M \end{cases} \quad (17)$$

Fig. 5 shows the results of calculations of $|S_{11}|$ of the line of Fig. 1 after 200 iterations ($t = 200\Delta t$) with the two types of excitation. The dashed line was obtained with δ type excitation, while the continuous line was obtained with the excitation of limited spectrum (with ω_c corresponding to $a/\lambda = 0.2$ and $M = 50$). When the computation is prolonged, the results obtained with the second type of pulse change very little while the ripples on the curve obtained with the δ type of excitation decrease slowly, and after about 1500 iterations the shape of the curve approaches the result obtained before with the second type of pulse.

The reason for this effect is that the FD approximation produces some resonances above the investigated frequency band. A δ pulse excites the circuit at these frequencies and produces a ringing-type response. Cutting off the high frequencies from the exciting pulse eliminates the effect.

However, it should be noted that when the resonances of the investigated circuit lie within the band of interest their effect cannot be eliminated by changing the exciting pulse spectrum. An example of such a circuit will be shown later in this paper.

We may conclude that since using a δ type of pulse simplifies the algorithm and speeds it up (because the Fourier transform of the source does not have to be computed), it is a reasonable choice in many cases. However if we consider a relatively resonance free band and the resonances are grouped outside that band, the time of computing can be brought substantially down by applying a pulse of the spectrum limited in such a way that the unwanted resonances are not excited.

C. Microstrip Circuits Analysis

As was already shown in [7], the FD-TD method is effective in arbitrarily shaped stripline circuit analysis. The fringing fields were included in the calculations by assuming that the circuit is bounded by a magnetic wall shifted by some distance from the real edge of the circuit. For a microstrip circuit a more complicated model is needed to describe the complicated nature of the fringing fields. Here is a proposal of such a model.

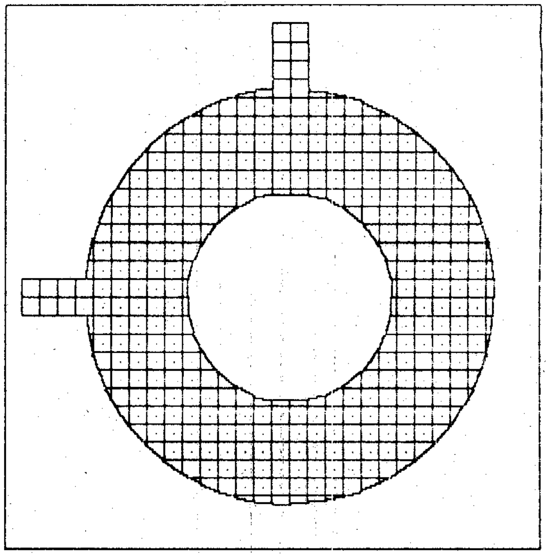

Fig. 6. A microstrip ring circuit as a grid of meshes.

Let us consider a microstrip line of width w. This line is characterized by its unit capacitance $C(w)$, unit inductance $L(w)$, and effective permittivity $\epsilon_{eff}(w)$ (with permeability $\mu = \mu_0$). Let us now imagine a line filled with a uniform dielectric characterized by $\epsilon' = \text{const}$ and $\mu' = \text{const}$ and having such properties that its unit capacitance C' and unit inductance L' obey the relations

$$C'(w + \Delta w) = C(w) \quad (18)$$

$$L'(w + \Delta w + \Delta w') = L(w). \quad (19)$$

In many practical cases it is possible to find such values of Δw and $\Delta w'$ that (18) and (19) are obeyed with good accuracy over a wide range of w. For example, for a duroid substrate of $\epsilon = 10\epsilon_0$ and height $h = 0.635$ mm, drawing the functions $C(w)$ and $L(w)$ from the closed-form expressions for Z_0 and ϵ_{eff} after [9] we find that (18) and (19) are obeyed with 1 percent accuracy in the range $2h < w < 10h$, assuming that $\Delta w = 1.05h$, $\Delta w' = h$, $\epsilon' = 10\epsilon_0$, and $\mu' = 0.93\mu_0$. For wider strips the error of this approximation rises only slightly. For very narrow strips it is bigger but can be corrected by adopting for the edges of these strips different values of Δw and $\Delta w'$.

We will now apply the FD-TD method to the analysis of a ring circuit already analyzed by another method by D'Inzeo *et al.* [10]. The circuit was built on the duroid substrate, had the dimensions of $r_{out} = 7$ mm, $r_{in} = 4$ mm, and was connected to two 50 Ω lines making a 90° angle. For FD-TD calculations we assumed the circuit as presented in Fig. 6. Its dimensions were obtained by shifting the circuit's boundary by $\Delta w/2 = 0.525h$ and it was assumed that $\epsilon' = 10\epsilon_0$ and $\mu' = 0.93\mu_0$. We applied the boundary matching procedure described in [7] but took into account additional inductance distributed along the

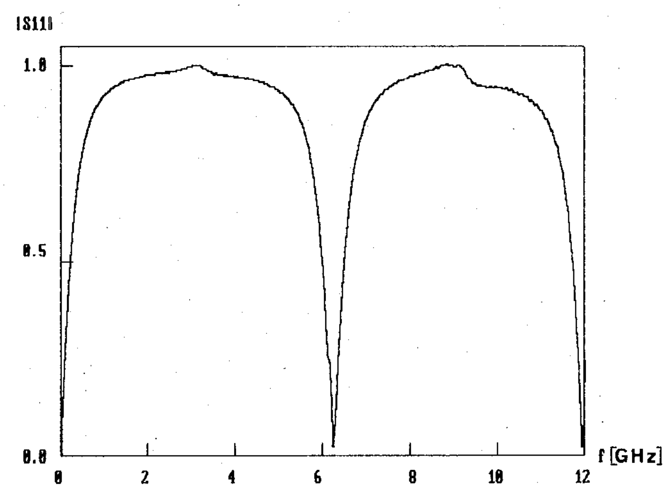

Fig. 7. $|S_{11}|$ versus frequency of the circuit of Fig. 6.

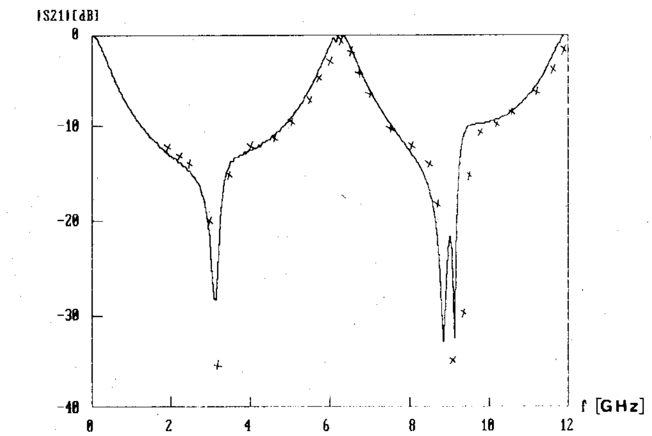

Fig. 8. $|S_{21}|$ versus frequency of the circuit of Fig. 6. —— calculations, × × × × × measurements after [10].

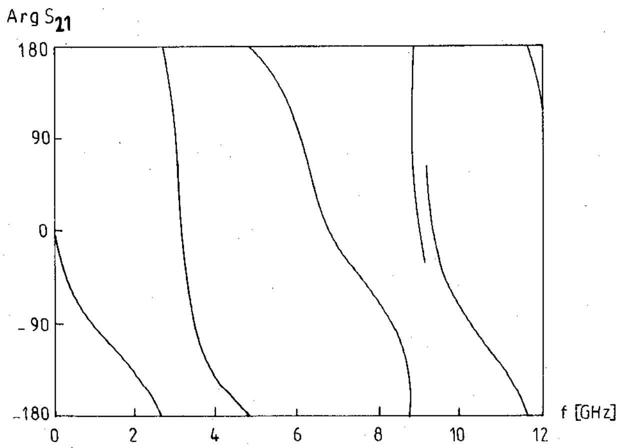

Fig. 9. Arg(S_{21}) versus frequency of the circuit of Fig. 6.

border. This corresponded to the inductance of a line of width $\Delta w'/2 = 0.5h$. The results of calculations of $|S_{11}|$, $|S_{21}|$, and Arg(S_{21}) are presented in Figs. 7, 8, and 9. In the case of $|S_{21}|$ they are compared with the measurements after [10]. The agreement is quite good. It has to be stressed that the method presented here does not assume

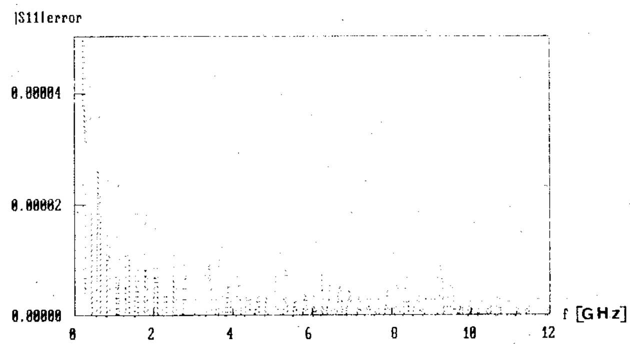

Fig. 10. Error in calculations of $|S_{11}|$ in Fig. 7 caused by a simulated low accuracy of the computer arithmetics equal to 10^{-6}.

any regularities of the circuit's shape. Thus it is a method for truly arbitrarily shaped circuits.

We may conclude that the presented example of the FD-TD method application to an arbitrarily shaped microstrip circuit is encouraging, but more work has to be done to check its value for various circuits, especially in the higher frequency band, when dispersion becomes important.

D. *Roundoff Errors*

In many programming languages there is a choice of the precision of the floating point arithmetic and it is important to know which precision to choose to keep the computer roundoff errors negligible while not boosting the memory requirements and the computing time. To check the level of the roundoff errors the calculations of the circuit of Fig. 6 were repeated with a simulated low computer precision of 10^{-6}. The difference between the results of $|S_{11}|$ obtained with full and with limited precision with δ-type excitation are presented in Fig. 10. It is seen that computer precision even as low as 10^{-6} introduces negligible errors into FD-TD calculations. This and other numerical experiments have shown that the error level increases with decreasing a/λ ratio (as seen in Fig. 10) and that it is slightly smaller for other than δ-type excitations but these dependencies have negligible effect due to generally low level of the roundoff errors. We may draw the conclusion that the FD-TD method is very resistant to roundoff errors and in most cases can be applied even with the lowest (4 bytes) precision of the floating point arithmetics used in personal computers.

E. *Algorithm Implementation*

The described problems of the FD-TD analysis were checked with a Pascal program prepared for an IBM PC AT computer. The program was written in a "user friendly" manner under the assumption that it will serve a variety of users.

As an example, the time needed in computing the S matrix of the circuit of Fig. 6 was about 20 min. for 1000 iterations. The results presented in Figs. 7–9 were obtained after 3000 iterations, but reasonably accurate results can be obtained even after 1000 iterations. In this case the

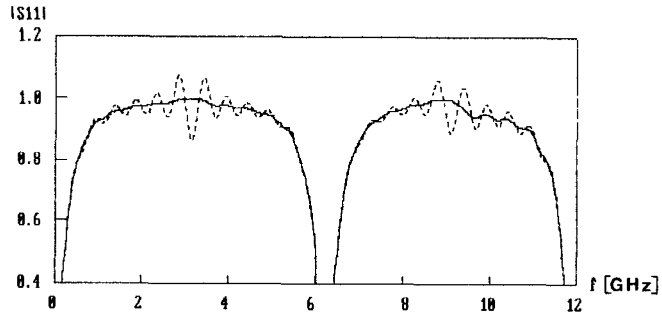

Fig. 11. $|S_{21}|$ of the circuit of Fig. 6 after 1000 iterations (dashed line) and the same result after smoothing procedure using the criterion of power conservation (continuous line).

functions obtained have some ripples (see, for example, the dotted line in Fig. 11) but these ripples can be eliminated using the criterion $|S_{11}|^2 + |S_{21}|^2 = 1$, valid for lossless circuits (see the continuous line in Fig. 11, which is practically the same as that of Fig. 7).

F. Comparison to Other Methods

As was mentioned in the introduction, the methods which can be assigned to the same class as possible competitors to the FD-TD method are the contour integral (CI) method [2], [13] and the transmission-line matrix (TLM) method [3], [11], [12].

In the contour integral method [2], the circuit's boundary is divided into a set of N elements of finite width. Using the properties of cylindrical waves for a particular frequency, we can obtain the relations between the voltages at the elements. This leads to a set of N linear equations to be solved.

When comparing the FD-TD and the CI methods we have to note first that their domains of application are not the same. The FD-TD method can be applied to circuits filled with nonuniform media and also to circuits with nonlinear elements (when restricted to sinusoidal excitation). These two classes of circuits cannot be treated by the CI method.

It is difficult to say which of the two methods needs less computer time. There are three reasons for this.

1) It depends on the type of circuit. Many factors contribute, for example,

- the computer time needed by the CI method depends on the length of the boundary line while the time of the FD-TD calculations depends on the surface of the circuit;
- for some circuits greater advantage may be taken from the flexibility of the CI method in applying varying boundary element length.
- the time of computing by the FD-TD method depends on the presence of high-Q resonances in the circuit.

2) Specialized hardware and software can bring a dramatic drop in computing time for either of the two methods, but the gain obtained with particular computer resources may be quite different for each of them.

3) In the CI method calculations are conducted for a particular frequency and have to be repeated from the very beginning when the circuit's parameters are to be calculated for another frequency. On the contrary, in the FD-TD method calculating the circuit's parameters for several hundred frequencies brings only a fractional increase in the computing time with respect to one-frequency calculations.

As an example, the computing time by both methods was compared for the circuit of Fig. 6 (but with no additional inductance along the border, a situation which cannot be handled by the CI method) on an IBM PC AT computer using standard library routines. In the time needed for calculating the wide-band frequency characteristics by the FD-TD method only a few (about five) frequency points could be calculated by the CI method.

An important advantage of the FD-TD method is that all algorithm operations have clear physical interpretations. That is why if a computing error occurs its cause can be quite easily spotted. This is in contrast to calculations by the CI method involving large-scale matrix operations which are very difficult to control.

The FD-TD method and the transmission-line matrix (TLM) method [3], [11] are similar in many respects. They both use a net of square meshes and both consider a pulse propagation in the net (although the algorithm of simulating the pulse propagation is different). Comparison of the FD-TD and the TLM methods has been discussed in recent publications [12], [16]. The general conclusion is that those methods are developed in a parallel way. Progress in one of them triggers development of the other. For example, to the author's knowledge the TLM method was not yet applied to S matrix calculations by modeling the circuit with matched input and output, but taking into account the discussion of this paper such an approach seems straightforward. Although the author found that in the considered application his version of the FD-TD method is faster and more convenient than the versions of the TLM known to him, he thinks of his work as a step forward in developing the entire group of wave-simulating methods, which includes the FD-TD and the TLM.

III. Conclusions

The paper has presented a version of the finite-difference time-domain method adapted to the needs of the S matrix calculations of two-dimensional microwave circuits. To allow direct S matrix calculations, matched loads and matched sources were modeled in the algorithm. This modeling gave good results over a wide frequency range (covering mesh size to wavelength ratios $0 < a/\lambda < 0.1$).

Pulse excitation was introduced in the method for fast calculations of frequency-dependent circuit characteristics. It was shown that the pulse excitation does not degrade the FD-TD accuracy. Different types of pulses were studied. In some cases a δ-type pulse is the most practical choice but in the cases when the circuit's main resonances are grouped outside the band of interest a pulse of limited spectrum can bring a substantial drop in the time of computing.

It was shown that the FD-TD method is not sensitive to the computer roundoff errors. Low precision of the floating point numbers may be chosen in computer programs to keep down the computer time and memory.

An analysis of an arbitrarily shaped microstrip circuit was presented as an example. Good agreement between the results of calculations and measurements was obtained but more work has to be done to check the applied fringing fields models for various circuits.

Implementation of the described method on an IBM PC AT computer shows that it can become a practical tool in engineering applications. In many cases it performs better than other widely used methods such as the contour integral method and the transmission-line matrix method.

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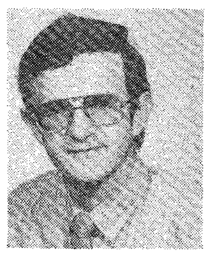

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